AS/A Level Mathematics (9709):

Integration

Lesson 2.5.1

Integration of Exponential and Logarithmic Functions

Revision Notes

1. Integrate: $\int e^{2x+3}dx$

Let $y = \int e^{2x+3}dx$ $\left[\because \int e^{ax+b}dx = \frac{1}{a}e^{ax+b} + c\right]$

$= \frac{1}{2}\left(e^{2x+3}\right) + c$

2. What is the integral of : $\int \sqrt{e^{x+1}}\,dx$**?**

Simplifying given expression as:

$$\int \sqrt{e^{x+1}}dx = \int \left(e^{x+1}\right)^{\frac{1}{2}} dx = \int e^{\left(\frac{x}{2}+\frac{1}{2}\right)} dx$$

$$\int e^{\left(\frac{x}{2}+\frac{1}{2}\right)} dx = \frac{1}{\left(\frac{1}{2}\right)}e^{\left(\frac{x}{2}+\frac{1}{2}\right)} + c \qquad \left[\because \int e^{ax+b}dx = \frac{1}{a}e^{ax+b} + c\right]$$

$$= 2e^{\left(\frac{x}{2}+\frac{1}{2}\right)} + c$$

3. Evaluate: $\int_0^1 e^{2x+3}dx$

First integrate without limits,

$$\int e^{2x+3}dx = \frac{1}{2}e^{2x+3} + c$$

Now use limits as:

$$\int_0^1 e^{2x+3}dx = \left[\frac{1}{2}e^{2x+3} + c\right]_0^1$$

$$= \left[\frac{1}{2}e^{(2\times1+3)} + c - \frac{1}{2}e^{(2\times0+3)} - c\right]$$

$$= \left[\frac{1}{2}e^{5} + c - \frac{1}{2}e^{3} - c\right]$$

$$= \left(\frac{1}{2}e^{5} - \frac{1}{2}e^{3}\right)$$

$$= \frac{1}{2}e^{3}\left(e^{2} - 1\right)$$

4. Find the exact integral value of: $\int_0^1 e^{-x}\left(1 + e^{x}\right)dx$

Simplifying the given expression

$$\int_0^1 e^{-x}\left(1 + e^{x}\right)dx = \int_0^1\left(e^{-x} + e^{-x}\cdot e^{x}\right)dx$$

$$= \int_0^1 (e^{-x} + 1)\,dx \qquad \left[\because e^{-x}\cdot e^{x} = e^{0} = 1\right]$$

$$= \int_0^1 e^{-x}dx + \int_0^1 1\,dx$$

Integrating and assigning the limits

$$\int_0^1 e^{-x}\left(1 + e^{x}\right) dx = \left[\frac{1}{(-1)} e^{-x}\right]_0^1 + [x]_0^1$$

$$= \left[- e^{-x}\right]_0^1 + [x]_0^1$$

$$= \left[- e^{-1} + e^{0}\right] + [1 - 0]$$

$$= 2 - e^{-1}$$

$$= \left(2 - \frac{1}{e}\right)$$

5. Integrate: $\int \frac{1}{3x} dx$

Formula: $\int \frac{1}{ax} dx = \frac{1}{a} ln|x| + c$

So, $\int \frac{1}{3x} dx = \frac{1}{3} \int \frac{1}{x} dx$

$$= \frac{1}{3} ln|x| + c$$

6. Integrate: $\int \frac{2}{2x+3} dx$

$$\int \frac{2}{2x+3} dx = ln|2x + 3| + c \quad \left[\because \int \frac{a}{ax+b} dx = ln|ax + b| + c\right]$$

7. Integrate: $\int \frac{1}{3x-7} dx$

$$\int \frac{1}{3x-7} dx = \frac{1}{3} \int \frac{3}{3x-7} dx \quad \left[\because \frac{d}{dx}(3x - 7) = 3\right]$$

$$= \frac{1}{3} ln|3x - 7| + c \left[\because \int \frac{f'(x)}{f(x)} dx = ln|f(x)| + c\right]$$

8. Evaluate: $\int_1^2 \frac{2x+x^2+5}{x}dx$

Separate the terms first

$$\int_1^2 \frac{2x+x^2+5}{x}dx = \int_1^2 \left(\frac{2x}{x} + \frac{x^2}{x} + \frac{5}{x}\right)dx$$

$$= \int_1^2 \left(2 + x + \frac{5}{x}\right)dx$$

$$= \int_1^2 2dx + \int_1^2 xdx + 5\int_1^2 \frac{1}{x}dx$$

$$= [2x]_1^2 + \left[\frac{x^2}{2}\right]_1^2 + 5[ln|x|]_1^2$$

Evaluate integral using limits

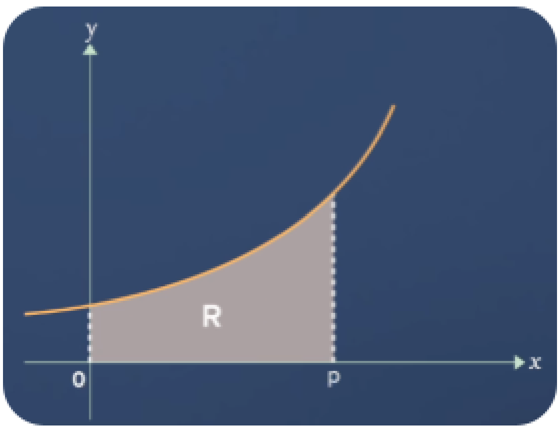
$$= [2 \times 2 - 2 \times 1] + \left[\frac{2^2}{2} - \frac{1^2}{2}\right] + 5[ln|2| - ln|1|]$$

$$= [4 - 2] + \left[2 - \frac{1}{2}\right] + 5[ln2 - ln1]$$

$$= 4 - \frac{1}{2} + 5ln\left(\frac{2}{1}\right) \qquad \left[\because ln2 - ln1 = ln\left(\frac{2}{1}\right)\right]$$

$$\int_1^2 \frac{2x+x^2+5}{x}dx = \frac{7}{2} + 5ln(2)$$

9. The diagram shows the curve $y = e^{2x}$. The shaded region R is bounded by the curve and by the lines $x = 0$, $y = 0$ and $x = p$. What will be the value of p if the area of R is equal to 5. Give your answer correct to 2 significant figures.



Given,

Equation of the curve: $y = e^{2x}$

Area $= \int_{0}^{p} e^{2x} dx$

$= \left[\frac{1}{2}e^{2x}\right]_0^p$

$= \frac{1}{2}e^{2p} - \frac{1}{2}$

It is given that Area of $R = 5$

Substituting Area $= 5$,

or, $\frac{1}{2}e^{2p} - \frac{1}{2} = 5$

or, $\frac{1}{2}e^{2p} = \frac{11}{2}$

or, $e^{2p} = 11$

or, $ln\left(e^{2p}\right) = ln(11)$

or, $2p = ln(11)$ $[\because ln(e) = 1]$

or, $p = \frac{ln(11)}{2} = 1.194$......

Therefore, $p = 1.2$(correct to 2 significant figures)